SAT MATH SECTION: CHEAT SHEET
FORMULAS, RULES, AND DEFINITIONS

Note: important formulas included at beginning of every test not included below

ALGEBRA:

ZERO PRODUCT PROPERTY: If $ab = 0$, then $a = 0$, $b = 0$, or $a = b = 0$.

ONE, ZERO, or INFINITE SOLUTIONS

SINGLE VARIABLE EQUATIONS:
If a single variable equation has ONE, ZERO or INFINITE solutions:

One solution	**Zero solutions**	**Infinite solutions**
You'll get a single x (or whatever variable is in the equation) value, (i.e. x=5 or y=2)	For a single variable equation to have no solution, all of the elements in the equation with the variable must cancel out, (i.e. if 2x is on one side of an equation and ax is on the other, a=2), and any remaining constants must create a statement that isn't true. If you get a statement that is never true (i.e. 5=9 or 0=4), you have no solutions.	For a single variable equation to have no solution, all of the elements in the equation with the variable must cancel out, (i.e. if 2x is on one side of an equation and ax is on the other, a=2), and any remaining constants must create a statement that is ALWAYS true, for example, if 3x+4k=ax+12, with variable x and constants k and a, has infinite solutions, 4k must =12. If you get two values that always equal each other (i.e. 3x=3x or 5=5), you have infinite solutions.

SYSTEMS OF EQUATIONS: To determine if a system has ONE, ZERO or INFINITE solutions you have three methods:

1. **Solve the system using elimination or substitution**

When you solve down a system of equations, one of three outcomes is possible as you run the algebra and create a final, simplified equation:

One solution	**Zero solutions**	**Infinite solutions**

If you get a single x or y value, (i.e. x=5 or y=2) you have one solution.	If you get a statement that is never true (i.e. 5=9 or 0=4), you have no solutions.	If you get two values that always equal each other (i.e. 3x=3x or 5=5), you have infinite solutions.

2. **Comparing equations:** You can alternatively put EVERY equation in slope intercept form that is part of your system of linear equations. Then look at each of the y=mx+b forms and compare the two slopes (i.e. the “m’s”) and y-intercepts (the “b’s”).

One solution	**Zero solutions**	**Infinite solutions**
Different slopes (intercepts don’t matter) On a graph this is two lines intersecting at a single point.	Same slopes (parallel lines), different y-intercepts On a graph this is two parallel lines.	Same slope, same intercept i.e. identical equations On a graph, this is the same line (i.e. both equations you have are actually identical but not “simplified” the same).

3. **Graphing in Desmos:** Put every equation in Desmos. See chart above.

SOLVING SYSTEMS OF EQUATIONS:

TIP: Look for shortcuts / use elimination on systems of equations questions to get what the question asks for rather than solve down for each individual variable. I.e. if the question asks for x+y, try to use elimination/adding or subtracting the whole equations to end up with x+y straightaway.

FOIL AND FACTORING:

- A **monomial** is a single product such as $4x, 7x^3,$ or $8n^2$.
- A **binomial** has two elements added together such as $4x+3$ or $5n^3+3n$.
- A **polynomial** has multiple elements added together such as $5n^3+3n^2+7n+2$ or $5n^3+3n^2+7n+2$ or $5x^2+2x+4$ $5x^2+2x+4$.
- Difference of Squares: The product of the difference $(a-b)$ and the sum ($a+b$) is equal to a squared minus b squared, $(a-b)(a+b)=a^2-b^2$
- Square of a Sum: $(a+b)^2=a^2+2ab+b^2$
- Square of a Difference: $(a-b)^2=a^2-2ab+b^2$

SLOPE FORMULA: For points (x_1, y_1) and (x_2, y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$.

STANDARD FORM SLOPE SHORTCUT: If $Ax + By = C$, slope is equal to $-\frac{A}{B}$

SLOPE-INTERCEPT FORM: $y = mx + b$
Where m is the **slope** of the line, and b is the **y-intercept** of the line at point $(0, b)$.

MIDPOINT FORMULA: The midpoint of two coordinate points (x_1, y_1) and (x_2, y_2) is:
$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$

$$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

DISTANCE FORMULA: Given two points, (x_1, y_1) and (x_2, y_2), the distance between them is:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

SPEED AND RATES DISTANCE FORMULA: $d = rt$
WORK FORMULA: $w = rt$
COMBINED WORK FORMULA: $w = (r_1 + r_2)t$

QUADRATICS AND POLYNOMIALS: Important- know how to find the vertex of ANY parabola!

- VERTEX FORM OF A QUADRATIC: the vertex form of a parabola is: $f(x) = a(x - h)^2 + k$
  - The vertex of the parabola in this form is (h, k)
  - When a is positive, the parabola opens upwards, and the minimum is (h, k).
  - When a is negative, the parabola opens downwards, and the minimum is (h, k).
  - Alternate form for parabolas with horizontal axis (RARE, unlikely to appear) $x = a(y - k)^2 + h$, positive a opens right-ward, negative a opens leftward

- FACTORED FORM: The factored form of a polynomial usually takes the form: $f(x) = a(x - n)(x - m)$
  - When a is positive, the parabola opens upwards.
  - When a is negative, the parabola opens downwards.
  - The graph crosses the x-axis at each point when a factored piece equals zero (the zeros). For example, at x=n, x-n equals 0, so the graph crosses through the point (n, 0) on the x-axis.

  In the case of a quadratic, which will typically have two roots (i.e. two factored pieces that include an x), the midpoint of 2 zeros is the x-value of the vertex. In the example above, the zeros are at x=n and x=m. So the x value of its vertex is the average of n and m.

- STANDARD FORM: the standard form of the parabola has the general form: $f(x) = ax^2 + bx + c$
  - $-\frac{b}{2a}$ is the x-value of the vertex. The y-value of the vertex can be found by plugging in this value for x and solving for y (or $f(x)$). Memorize this!
  - Assuming a vertically oriented parabola, the vertex is always either the maximum or the minimum of the graph. (remember max/min is always the y-value!)
  - When a is positive, the parabola opens upwards.
  - When a is negative, the parabola opens downwards.
  - The sum of the two roots is $-\frac{b}{a}$ (memorize if aiming for 750+)
  - The product of the two roots is $\frac{c}{a}$ (memorize if aiming for 750+)

- QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Memorize this. Also, we recommend you program it into your calculator. (See TI 84 programs for download in the online course or google our YouTube video on downloading programs TI 84)

- FINDING SOLUTIONS USING THE DISCRIMINANT: Given that $f(x) = ax^2 + bx + c$, the discriminant is defined as $b^2 - 4ac$ (the piece under the radical in the quadratic equation):
  - When this value is positive, there are two real roots
  - When this value is 0, there is one real root
  - When this value is negative, there are no real roots but two imaginary roots

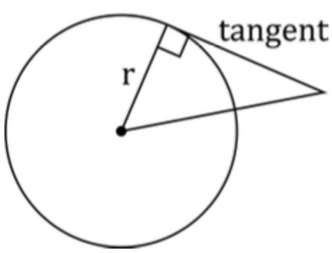
RADICAL AND EXPONENT RULES:

EXPONENT RULES	RADICAL RULES
Zero Power Rule: $a^0 = 1$	Fractional exponent conversion: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$
Negative exponent rule: $a^{-b} = \frac{1}{a^b} a$	Product of radicals: $\sqrt[c]{a}\sqrt[c]{b} = \sqrt[c]{ab}$
Power of a power: $(a^b)^c = a^{bc}$	
Product of powers: $(a^b)(a^c) = a^{c+b}$	
Power of a product: $(ab)^c = a^c b^c$	

Power of a quotient: $\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$	
Quotient of powers: $\frac{a^b}{a^c} = a^{b-c}$	

CIRCLE THEOREMS:

- **Circle Equation**: $(x-h)^2 + (y-k)^2 = r^2$ with center point at (h, k)
- A **tangent line** to a circle forms a 90° angle with the radius that touches the tangent point

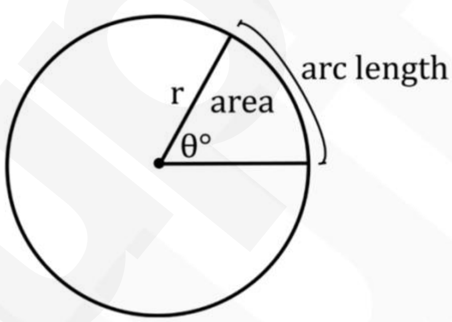


When you see the word TANGENT in a problem involving a circle:

- **Draw a radius** from the center to the tangent point(s).
- **Mark the angle** formed by a radius and tangent line as 90 degrees.
- **Look for right triangles** or draw any necessary additional lines to create them.
- **Use the Pythagorean Theorem, SOH CAH TOA, or other rules** you know as appropriate to solve.

Angle to Arc Length Equations: Central angles are proportional to the arc length they intercept, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{central\ angle\ measure}{total\ measure\ of\ angle\ in\ a\ circle} = \frac{arc\ length}{total\ circumference}$$



$$\frac{\theta°}{360°} = \frac{arc\ length}{circumference} = \frac{sector\ area}{total\ area}$$

DEGREES VERSION OF EQUATION:

- More specifically, this means if a circle has a central angle a, radius r, and arc length L:

$$\frac{a^\circ}{360^\circ} = \frac{L}{2\pi r}$$

RADIANS VERSION OF EQUATION:

- For any arc on a circle with a central angle (in radians) of θ, where r is the radius of the circle, the length L of the arc is given by:

$\frac{\theta}{2\pi} = \frac{L}{2\pi r}$ which simplifies to a short handy equation that's a good idea to memorize: $L = \theta r$.

Angle to Sector Area Equations: Central angles are proportional to the area of the sector they include, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{\textit{central angle measure}}{\textit{total measure of angle in a circle}} = \frac{\text{sector area}}{\text{total circle area}}$$

DEGREES VERSION OF EQUATION:

- For any sector on a circle with a central angle (in degrees) of a, where r is the radius of the circle, and the area is A:

$$\frac{a^\circ}{360^\circ} = \left(\frac{A}{\pi r^2}\right).$$

RADIANS VERSION OF EQUATION:

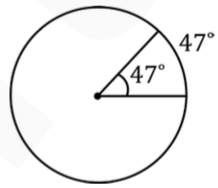
- For any sector in a circle with a central angle (in radians) of θ, where r is the radius of the circle, and the area A of that circle:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

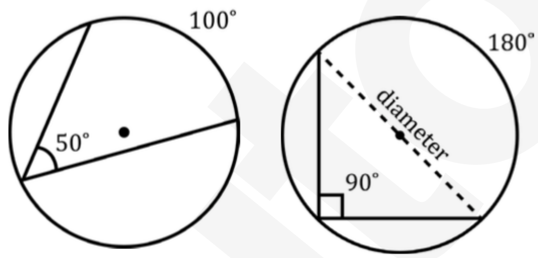
This simplifies to a shorter equation, which may merit memorizing:

$$A = \frac{1}{2}\theta r^2$$

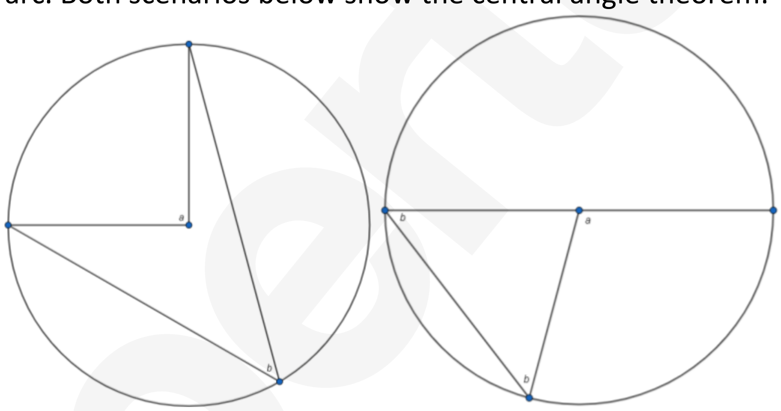
Central angles (angles formed by two radii that touch the circle in two places) in a circle are always the same degree measure as their corresponding arcs. For example, in the following picture, the arc is $47°$ and so is the central angle:



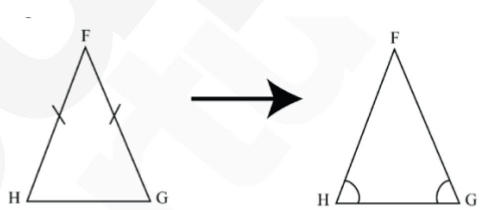
Inscribed angles (formed by two chords) are **half the corresponding arc measure**. For example, a 50 degree angle always opens to a 100 degree arc. A 90 degree angle always opens to a 180 degree arc. In fact, this is the example I use to remember this rule if I forget it, because it's so common to see in problems.



- **Inscribed Angle Theorem**: States that an angle (portrayed as b below) inscribed in a circle is equal to half of the central angle (portrayed as a below) that subtends the same arc. Both scenarios below show the central angle theorem:



Isosceles Triangle:
If two sides of a triangle are equal, their opposite angles are congruent.



PYTHAGOREAN TRIPLES

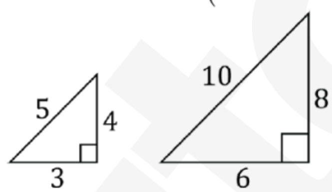
A Pythagorean triple consists of three positive integers a, b, and c such that $a^2+b^2=c^2$. Such a triple is commonly written (a,b,c). Some well-known examples are:

$$(3,4,5),(5,12,13),(8,15,17), \& (7,24,25).$$

TIMING TIP: **MEMORIZE THESE!** Knowing these will vastly speed up your math performance!

MULTIPLES OF PYTHAGOREAN TRIPLES

If (a,b,c) is a Pythagorean triple, then so is (ka,kb,kc) for any positive integer k



Trigonometry:

SOHCAHTOA: For any angle θ in a right triangle, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacient}{hypotenuse}$ and $\tan\theta = \frac{opposite}{hypotenuse}$.

Advanced Math:

- **Function Notation: (many of these are unlikely to appear, but you should understand how function notation works)**
- $(f \circ g)(x)$ means $f(g(x))$
- $(f * g)(x)$, $(f \cdot g)(x)$, $f(x)\cdot g(x)$, and $fg(x)$ mean multiply $f(x)\ and\ g(x)$
- $\left(\frac{f}{g}\right)(x)$ means $\frac{f(x)}{g(x)}$

Problem Solving and Data Analysis:

General Statistics:

- Average or Mean: Sum of terms divided by number of terms
- Median: The numerical middle value of a set of terms
- Mode: The most common term in a set of terms
- Range: The largest value in a set minus the smallest value in the set

Graphs/Plots:

- **Line of Best Fit:** A line that best represents the data on a scatterplot. It can be used to estimate the value of points not on the plot itself. **TIP:** pluck points and use a calculator program to find y=mx+b on scatterplots OR use the TABLE function in Desmos.

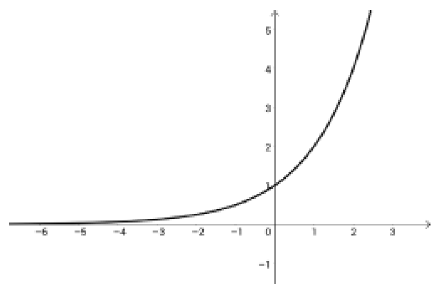
GRAPH BEHAVIOR:

Types of graphs:

Graph of an exponential function:

Standard Form: $f(x) = a^x$

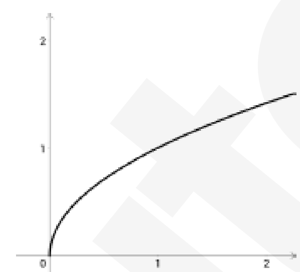
Example: $y = 2^x$



Graph of a square root:

Standard Form: $a\sqrt{x}$

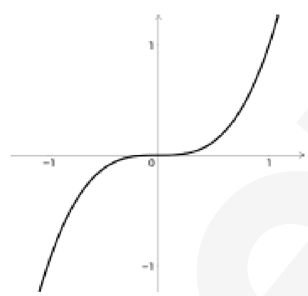
Example (Parent Graph): $y = \sqrt{x}$



Graph of a cubic function:

Vertex Form: $y = a(x-h)^3 + k$

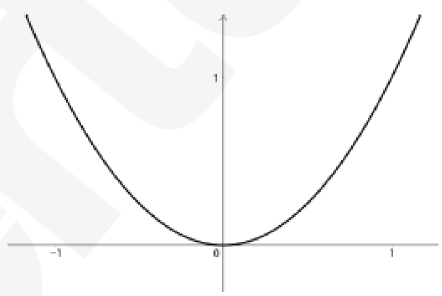
Example (Parent Graph): $y = x^3$



Graph of a parabola (quadratic function):

Vertex Form: $f(x) = a(x-h)^2 + k$

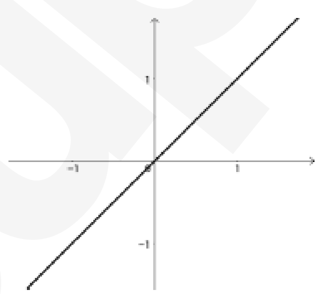
Example (Parent Graph): $y = x^2$



Graph of a linear equation:

Slope-Intercept Form: $y = mx + b$

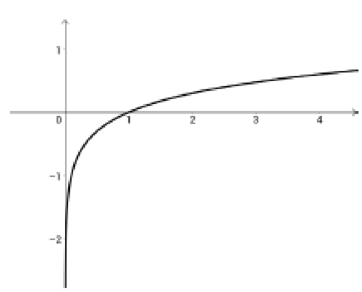
Example (Parent form): $y = x$



Graph of a logarithmic function:

Standard Form: $y = \log_a x$

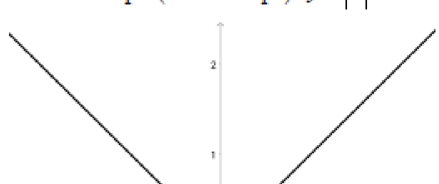
Example: $y = \log_{10} x$



Graph of an absolute value function:

Vertex Form: $y = a|x-h| + k$

Example (Parent Graph): $y = |x|$



- Horizontal Shift: For ALL functions, if you replace all instances of x in a function with "$(x-h)$" you'll find that the graph moves "h" units to the right.
- Vertical Shift: If you replace all instances of y in a function with "$(y-k)$" you'll find that the graph moves "k" units upward.

END BEHAVIOR:

Degree: Even	**Degree: Even**
Leading Coefficient: positive	Leading Coefficient: negative
End Behavior: $f(x)$ approaches $+\infty$ at both ends of the graph (upward facing)	End Behavior: $f(x)$ approaches $-\infty$ at both ends of the graph (downward facing)
Domain: all reals	Domain: all reals
Range: all reals ≥ maximum	Range: all reals ≤ maximum
Example: $y=x^2$	Example: $y=-2x^6+3x^5+4x$

Degree: odd	**Degree: odd**
Leading Coefficient: positive	Leading Coefficient: negative
End Behavior: At graph left, $f(x)\to-\infty$. At graph right, $f(x)\to+\infty$ (upward sloping)	End Behavior: At graph left, $f(x)\to+\infty$. At graph right, $f(x)\to-\infty$ (downward sloping)
Domain: all reals	Domain: all reals
Range: all reals	Range: all reals
Example: $f(x)=x^3$	Example: $f(x)=-3x^5-2x$

Desmos Fluency:
+Be sure you know how to use a CHART to find an equation from values of a graph (regression).

+Be sure you know how to use a SLIDER.
+You can only use a slider generally with two variable equations. If necessary, add a 0x term to allow you to use a slider with a single variable.

+Beware of turning to Desmos when a faster way exists.

+Beware of using Desmos on open answer questions or questions with complex answer choices (including radicals, weird fractions, etc.)