

SAT MATH SECTION: CHEAT SHEET FORMULAS, RULES, AND DEFINITIONS

Note: important formulas included at beginning of every test not included below

ALGEBRA:

ZERO PRODUCT PROPERTY: If $ab = 0$, then $a = 0$, $b = 0$, or $a = b = 0$.

SYSTEMS OF EQUATIONS:

When you solve down a system of equations, one of three outcomes is possible as you run the algebra and create a final, simplified equation:

| One solution | Zero solutions | Infinite solutions |
|---|--|---|
| If you get a single x or y value, (i.e. $x=5$ or $y=2$) you have one solution. | If you get a statement that is never true (i.e. $5=9$ or $0=4$), you have no solutions. | If you get two values that always equal each other (i.e. $3x=3x$ or $5=5$), you have infinite solutions. |
| Comparing equations: You can alternatively put EVERY equation in slope intercept form that is part of your system of linear equations. Then look at each of the $y=mx+b$ forms and compare the two slopes (i.e. the "m's") and y-intercepts (the "b's"). | | |
| One solution | Zero solutions | Infinite solutions |
| Different slopes (intercepts don't matter) On a graph this is two lines intersecting at a single point. | Same slopes (parallel lines), different y-intercepts On a graph this is two parallel lines. | Same slope, same intercept i.e. identical equations On a graph, this is the same line (i.e. both equations you have are actually identical but not "simplified" the same). |

TIP: Look for shortcuts / use elimination on systems of equations questions to get what the question asks for rather than solve down for each individual variable. I.e. if the question asks for $x+y$, try to use elimination/adding or subtracting the whole equations to end up with $x+y$ straightaway.

FOIL AND FACTORING:

- A **monomial** is a single product such as $4x$, $7x^3$, or $8n^2$.
- A **binomial** has two elements added together such as $4x+3$ or $5n^3+3n$.
- A **polynomial** has multiple elements added together such as $5n^3+3n^2+7n+2$ or $5n^3+3n^2+7n+2$ or $5x^2+2x+4$ or $5x^2+2x+4$.

- Difference of Squares: The product of the difference $(a - b)$ and the sum $(a + b)$ is equal to a squared minus b squared, $(a - b)(a + b) = a^2 - b^2$
- Square of a Sum: $(a + b)^2 = a^2 + 2ab + b^2$
- Square of a Difference: $(a - b)^2 = a^2 - 2ab + b^2$

SLOPE FORMULA: For points (x_1, y_1) and (x_2, y_2) , $m = \frac{y_2 - y_1}{x_2 - x_1}$.

STANDARD FORM SLOPE: If $Ax + By = C$, slope is equal to $-\frac{A}{B}$

SLOPE-INTERCEPT FORM: $y = mx + b$

Where m is the **slope** of the line, and b is the **y-intercept** of the line at point $(0, b)$.

MIDPOINT FORMULA: The midpoint of two coordinate points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

DISTANCE FORMULA: Given two points, (x_1, y_1) and (x_2, y_2) , the distance between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

SPEED AND RATES DISTANCE FORMULA: $d = rt$

WORK FORMULA: $w = rt$

COMBINED WORK FORMULA: $w = (r_1 + r_2)t$

QUADRATICS AND POLYNOMIALS: Important- know how to find the vertex of ANY parabola!

- VERTEX FORM OF A QUADRATIC: the vertex form of a parabola is: $f(x) = a(x - h)^2 + k$
 - The vertex of the parabola in this form is (h, k)
 - When a is positive, the parabola opens upwards, and the minimum is (h, k) .
 - When a is negative, the parabola opens downwards, and the minimum is (h, k) .
 - Alternate form for parabolas with horizontal axis (RARE, unlikely to appear) $x = a(y - k)^2 + h$, positive a opens right-ward, negative a opens leftward
- FACTORED FORM: The factored form of a polynomial usually takes the form:
 $f(x) = a(x - n)(x - m)$
 - When a is positive, the parabola opens upwards.
 - When a is negative, the parabola opens downwards.
 - The graph crosses the x-axis at each point when a factored piece equals zero (the zeros). For example, at $x=n$, $x-n$ equals 0, so the graph crosses through the point $(n, 0)$ on the x-axis.

In the case of a quadratic, which will typically have two roots (i.e. two factored pieces that include an x), the midpoint of 2 zeros is the x -value of the vertex. In the example above, the zeros are at $x=n$ and $x=m$. So the x value of its vertex is the average of n and m .

- **STANDARD FORM:** the standard form of the parabola has the general form: $f(x) = ax^2 + bx + c$
 - $-\frac{b}{2a}$ is the x -value of the vertex. The y -value of the vertex can be found by plugging in this value for x and solving for y (or $f(x)$). Memorize this!
 - Assuming a vertically oriented parabola, the vertex is always either the maximum or the minimum of the graph. (remember max/min is always the y -value!)
 - When a is positive, the parabola opens upwards.
 - When a is negative, the parabola opens downwards.
 - The sum of the two roots is $-\frac{b}{a}$ (not necessary to know, but helpful)
 - The product of the two roots is $\frac{c}{a}$ (not necessary to know, but occasionally helpful)
- **QUADRATIC FORMULA:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Memorize this. Also, we recommend you program it into your calculator. (See TI 84 programs for download in the online course)
- **FINDING SOLUTIONS USING THE DISCRIMINANT:** Given that $f(x) = ax^2 + bx + c$, the discriminant is defined as $b^2 - 4ac$ (the piece under the root in the quadratic equation):
 - When this value is positive, there are two real roots
 - When this value is 0, there is one real root
 - When this value is negative, there are no real roots but two imaginary roots

RADICAL AND EXPONENT RULES:

| EXPONENT RULES | RADICAL RULES |
|--|---|
| Zero Power Rule: $a^0 = 1$ | Fractional exponent conversion: $\sqrt[b]{a^b} = a^{\frac{b}{b}}$ |
| Negative exponent rule: $a^{-b} = \frac{1}{a^b}$ | Product of radicals: $\sqrt[a]{a} \sqrt[b]{b} = \sqrt[ab]{ab}$ |
| Power of a power: $(a^b)^c = a^{bc}$ | |

| | |
|---|--|
| Product of powers: $(ab)^c = a^{b+c}$ | |
| Power of a product: $(ab)^c = a^c b^c$ | |
| Power of a quotient: $\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$ | |
| Quotient of powers: $\frac{a^b}{a^c} = a^{b-c}$ | |

CIRCLE THEOREMS:

- Circle Equation: $(x - h)^2 + (y - k)^2 = r^2$ with center point at (h, k)

Angle to Arc Length Equations: Central angles are proportional to the arc length they intercept, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{\text{central angle measure}}{\text{total measure of angle in a circle}} = \frac{\text{arc length}}{\text{total circumference}}$$

DEGREES VERSION OF EQUATION:

- More specifically, this means if a circle has a central angle a , radius r , and arc length L :

$$\frac{a^\circ}{360^\circ} = \frac{L}{2\pi r}$$

RADIANS VERSION OF EQUATION:

- For any arc on a circle with a central angle (in radians) of θ , where r is the radius of the circle, the length L of the arc is given by:

$$\frac{\theta}{2\pi} = \frac{L}{2\pi r} \text{ which simplifies to a short handy equation that's a good idea to memorize: } L = \theta r .$$

Angle to Sector Area Equations: Central angles are proportional to the area of the sector they include, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{\text{central angle measure}}{\text{total measure of angle in a circle}} = \frac{\text{sector area}}{\text{total circle area}}$$

DEGREES VERSION OF EQUATION:

- For any sector on a circle with a central angle (in degrees) of a , where r is the radius of the circle, and the area is A :

$$\frac{a^\circ}{360^\circ} = \left(\frac{A}{\pi r^2} \right).$$

RADIANS VERSION OF EQUATION:

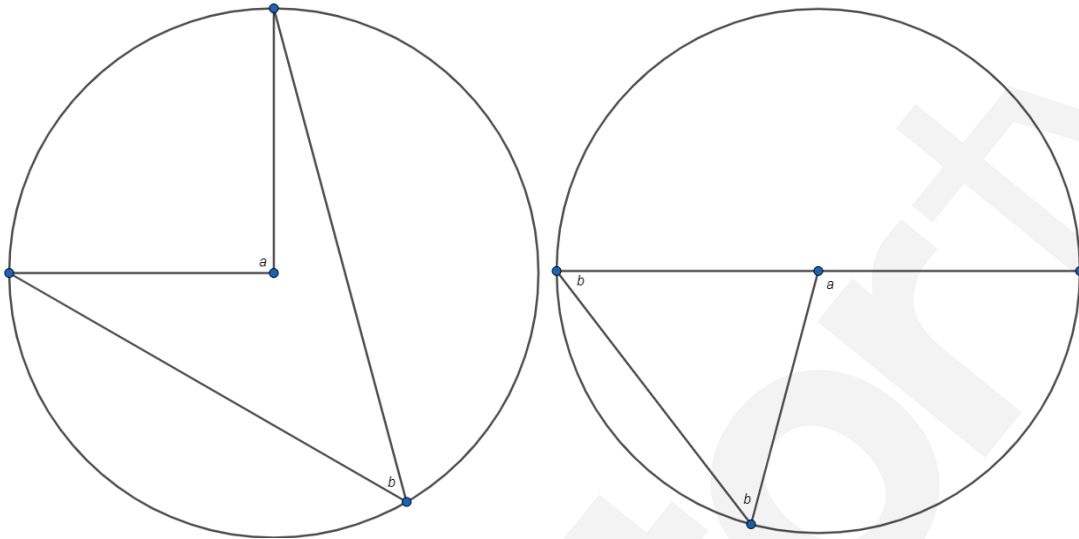
- For any sector in a circle with a central angle (in radians) of θ , where r is the radius of the circle, and the area A of that circle:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

This simplifies to a shorter equation, which may merit memorizing:

$$A = \frac{1}{2} \theta r^2$$

- Inscribed Angle Theorem: States that an angle (portrayed as b below) inscribed in a circle is equal to half of the central angle (portrayed as a below) that subtends the same arc. Both scenarios below show the central angle theorem:



- **Trigonometry:**

SOHCAHTOA: For any angle θ in a right triangle, $\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$, $\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$
 and $\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$.

Passport to Advanced Math:

- **Function Notation:** (many of these are unlikely to appear, but you should understand how function notation works)
- $(f \circ g)(x)$ means $f(g(x))$
- $(f * g)(x)$ and $fg(x)$ mean $f(x) * g(x)$
- $\left(\frac{f}{g}\right)(x)$ means $\frac{f(x)}{g(x)}$

Problem Solving and Data Analysis:

General Statistics:

- Average: Sum of terms divided by number of terms
- Median: The numerical middle value of a set of terms
- Mode: The most common term in a set of terms

Graphs/Plots:

- **Line of Best Fit:** A line that best represents the data on a scatterplot. It can be used to estimate the value of points not on the plot itself. **TIP:** pluck points and use a calculator program to find $y=mx+b$ on scatterplots!

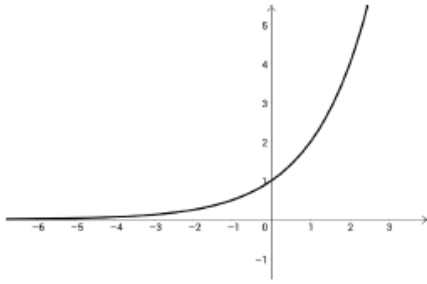
GRAPH BEHAVIOR:

Types of graphs:

Graph of an exponential function:

Standard Form: $f(x) = a^x$

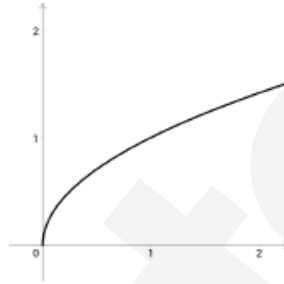
Example: $y = 2^x$



Graph of a square root:

Standard Form: $a\sqrt{x}$

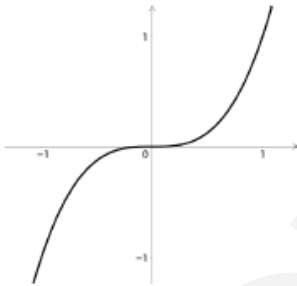
Example (Parent Graph): $y = \sqrt{x}$



Graph of a cubic function:

Vertex Form: $y = a(x-h)^3 + k$

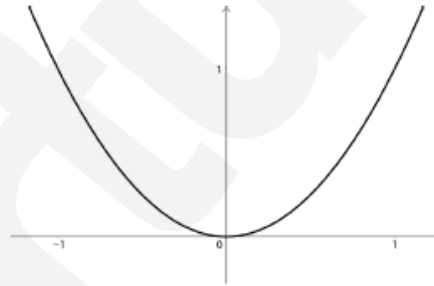
Example (Parent Graph): $y = x^3$



Graph of a parabola (quadratic function):

Vertex Form: $f(x) = a(x-h)^2 + k$

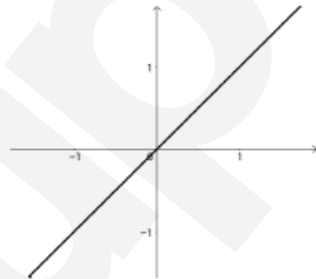
Example (Parent Graph): $y = x^2$



Graph of a linear equation:

Slope-Intercept Form: $y = mx + b$

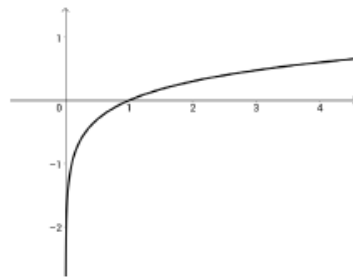
Example (Parent form): $y = x$



Graph of a logarithmic function:

Standard Form: $y = \log_a x$

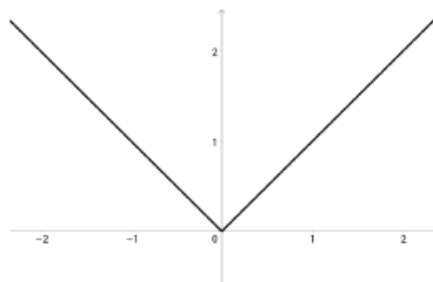
Example: $y = \log_{10} x$



Graph of an absolute value function:

Vertex Form: $y = a|x-h| + k$

Example (Parent Graph): $y = |x|$



- Horizontal Shift: For ALL functions, if you replace all instances of x in a function with " $(x - h)$ " you'll find that the graph moves "h" units to the right.
- Vertical Shift: If you replace all instances of y in a function with " $(y - k)$ " you'll find that the graph moves "k" units upward.

END BEHAVIOR:

| Degree: Even | Degree: Even |
|---|---|
| Leading Coefficient: positive | Leading Coefficient: negative |
| End Behavior: $f(x)$ approaches $+\infty$ at both ends of the graph (upward facing) | End Behavior: $f(x)$ approaches $-\infty$ at both ends of the graph (downward facing) |
| Domain: all reals | Domain: all reals |
| Range: all reals \geq maximum | Range: all reals \leq maximum |
| Example: $y = x^2$ | Example: $y = -2x^6 + 3x^5 + 4x$ |

| Degree: odd | Degree: odd |
|---|---|
| Leading Coefficient: positive | Leading Coefficient: negative |
| End Behavior: At graph left, $f(x) \rightarrow -\infty$. At graph right, $f(x) \rightarrow +\infty$ (upward sloping) | End Behavior: At graph left, $f(x) \rightarrow +\infty$. At graph right, $f(x) \rightarrow -\infty$ (downward sloping) |
| Domain: all reals | Domain: all reals |
| Range: all reals | Range: all reals |
| Example: $f(x) = x^3$ | Example: $f(x) = -3x^5 - 2x$ |