

## SAT MATH SECTION: CHEAT SHEET FORMULAS, RULES, AND DEFINITIONS

Note: important formulas included at beginning of every test not included below

### ALGEBRA:

ZERO PRODUCT PROPERTY: If  $ab = 0$ , then  $a = 0$ ,  $b = 0$ , or  $a = b = 0$ .

### SYSTEMS OF EQUATIONS:

When you solve down a system of equations, one of three outcomes is possible as you run the algebra and create a final, simplified equation:

| One solution                                                                                                                                                                                                                                                    | Zero solutions                                                                           | Infinite solutions                                                                                                     |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| If you get a single x or y value, (i.e. $x=5$ or $y=2$ ) you have one solution.                                                                                                                                                                                 | If you get a statement that is never true (i.e. $5=9$ or $0=4$ ), you have no solutions. | If you get two values that always equal each other (i.e. $3x=3x$ or $5=5$ ), you have infinite solutions.              |
| <b>Comparing equations:</b> You can alternatively put EVERY equation in slope intercept form that is part of your system of linear equations. Then look at each of the $y=mx+b$ forms and compare the two slopes (i.e. the "m's") and y-intercepts (the "b's"). |                                                                                          |                                                                                                                        |
| One solution                                                                                                                                                                                                                                                    | Zero solutions                                                                           | Infinite solutions                                                                                                     |
| Different slopes (intercepts don't matter)                                                                                                                                                                                                                      | Same slopes (parallel lines), different y-intercepts                                     | Same slope, same intercept i.e. identical equations                                                                    |
| On a graph this is two lines intersecting at a single point.                                                                                                                                                                                                    | On a graph this is two parallel lines.                                                   | On a graph, this is the same line (i.e. both equations you have are actually identical but not "simplified" the same). |

**TIP: Look for shortcuts / use elimination** on systems of equations questions to get what the question asks for rather than solve down for each individual variable. I.e. if the question asks for  $x+y$ , try to use elimination/adding or subtracting the whole equations to end up with  $x+y$  straightaway.

### FOIL AND FACTORING:

- A **monomial** is a single product such as  $4x$ ,  $7x^3$ , or  $8n^2$ .
- A **binomial** has two elements added together such as  $4x+3$  or  $5n^3+3n$ .
- A **polynomial** has multiple elements added together such as  $5n^3+3n^2+7n+2$  or  $5x^3+3n^2+7n+2$  or  $5x^2+2x+4$  or  $5x^2+2x+4$ .

- Difference of Squares: The product of the difference  $(a - b)$  and the sum  $(a + b)$  is equal to  $a$  squared minus  $b$  squared,  $(a - b)(a + b) = a^2 - b^2$
- Square of a Sum:  $(a + b)^2 = a^2 + 2ab + b^2$
- Square of a Difference:  $(a - b)^2 = a^2 - 2ab + b^2$

SLOPE FORMULA: For points  $(x_1, y_1)$  and  $(x_2, y_2)$ ,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .

STANDARD FORM SLOPE: If  $Ax + By = C$ , slope is equal to  $-\frac{A}{B}$

SLOPE-INTERCEPT FORM:  $y = mx + b$

Where  $m$  is the **slope** of the line, and  $b$  is the **y-intercept** of the line at point  $(0, b)$ .

MIDPOINT FORMULA: The midpoint of two coordinate points  $(x_1, y_1)$  and  $(x_2, y_2)$  is:

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

DISTANCE FORMULA: Given two points,  $(x_1, y_1)$  and  $(x_2, y_2)$ , the distance between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

SPEED AND RATES DISTANCE FORMULA:  $d = rt$

WORK FORMULA:  $w = rt$

COMBINED WORK FORMULA:  $w = (r_1 + r_2)t$

QUADRATICS AND POLYNOMIALS: Important- know how to find the vertex of ANY parabola!

- VERTEX FORM OF A QUADRATIC: the vertex form of a parabola is:  $f(x) = a(x - h)^2 + k$ 
  - The vertex of the parabola in this form is  $(h, k)$
  - When  $a$  is positive, the parabola opens upwards, and the minimum is  $(h, k)$ .
  - When  $a$  is negative, the parabola opens downwards, and the minimum is  $(h, k)$ .
  - Alternate form for parabolas with horizontal axis (RARE, unlikely to appear)  $x = a(y - k)^2 + h$ , positive  $a$  opens right-ward, negative  $a$  opens leftward
- FACTORED FORM: The factored form of a polynomial usually takes the form:  $f(x) = a(x - n)(x - m)$ 
  - When  $a$  is positive, the parabola opens upwards.
  - When  $a$  is negative, the parabola opens downwards.
  - The graph crosses the x-axis at each point when a factored piece equals zero (the zeros). For example, at  $x=n$ ,  $x-n$  equals 0, so the graph crosses through the point  $(n, 0)$  on the x-axis.

In the case of a quadratic, which will typically have two roots (i.e. two factored pieces that include an  $x$ ), the midpoint of 2 zeros is the  $x$ -value of the vertex. In the example above, the zeros are at  $x=n$  and  $x=m$ . So the  $x$  value of its vertex is the average of  $n$  and  $m$ .

- **STANDARD FORM:** the standard form of the parabola has the general form:  $f(x) = ax^2 + bx + c$ 
  - $-\frac{b}{2a}$  is the  $x$ -value of the vertex. The  $y$ -value of the vertex can be found by plugging in this value for  $x$  and solving for  $y$  (or  $f(x)$ ). Memorize this!
  - Assuming a vertically oriented parabola, the vertex is always either the maximum or the minimum of the graph. (remember max/min is always the  $y$ -value!)
  - When  $a$  is positive, the parabola opens upwards.
  - When  $a$  is negative, the parabola opens downwards.
  - The sum of the two roots is  $-\frac{b}{a}$  (not necessary to know, but helpful)
  - The product of the two roots is  $\frac{c}{a}$  (not necessary to know, but occasionally helpful)
- **QUADRATIC FORMULA:**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  Memorize this. Also, we recommend you program it into your calculator. (See TI 84 programs for download in the online course)
- **FINDING SOLUTIONS USING THE DISCRIMINANT:** Given that  $f(x) = ax^2 + bx + c$ , the discriminant is defined as  $b^2 - 4ac$  (the piece under the root in the quadratic equation):
  - When this value is positive, there are two real roots
  - When this value is 0, there is one real root
  - When this value is negative, there are no real roots but two imaginary roots

#### RADICAL AND EXPONENT RULES:

| EXPONENT RULES                                   | RADICAL RULES                                                               |
|--------------------------------------------------|-----------------------------------------------------------------------------|
| Zero Power Rule: $a^0 = 1$                       | Fractional exponent conversion: $\sqrt[b]{a^b} = a^{\frac{b}{b}} = a^1 = a$ |
| Negative exponent rule: $a^{-b} = \frac{1}{a^b}$ | Product of radicals: $\sqrt[a]{a} \sqrt[b]{b} = \sqrt[ab]{ab}$              |
| Power of a power: $(a^b)^c = a^{bc}$             |                                                                             |

|                                                                     |  |
|---------------------------------------------------------------------|--|
| Product of powers: $(ab)^c = a^{b+c}$                               |  |
| Power of a product: $(a^b)^c = a^c b^c$                             |  |
| Power of a quotient: $\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$ |  |
| Quotient of powers: $\frac{a^b}{a^c} = a^{b-c}$                     |  |

## CIRCLE THEOREMS:

- Circle Equation:  $(x-h)^2 + (y-k)^2 = r^2$  with center point at  $(h, k)$

**Angle to Arc Length Equations:** Central angles are proportional to the arc length they intercept, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{\text{central angle measure}}{\text{total measure of angle in a circle}} = \frac{\text{arc length}}{\text{total circumference}}$$

**DEGREES VERSION OF EQUATION:**

- More specifically, this means if a circle has a central angle  $a$ , radius  $r$ , and arc length  $L$ :

$$\frac{a^\circ}{360^\circ} = \frac{L}{2\pi r}$$

**RADIANS VERSION OF EQUATION:**

- For any arc on a circle with a central angle (in radians) of  $\theta$ , where  $r$  is the radius of the circle, the length  $L$  of the arc is given by:

$$\frac{\theta}{2\pi} = \frac{L}{2\pi r} \text{ which simplifies to a short handy equation that's a good idea to memorize: } L = \theta r .$$

**Angle to Sector Area Equations:** Central angles are proportional to the area of the sector they include, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{\text{central angle measure}}{\text{total measure of angle in a circle}} = \frac{\text{sector area}}{\text{total circle area}}$$

**DEGREES VERSION OF EQUATION:**

- For any sector on a circle with a central angle (in degrees) of  $a$ , where  $r$  is the radius of the circle, and the area is  $A$ :

$$\frac{a^\circ}{360^\circ} = \left( \frac{A}{\pi r^2} \right).$$

**RADIANS VERSION OF EQUATION:**

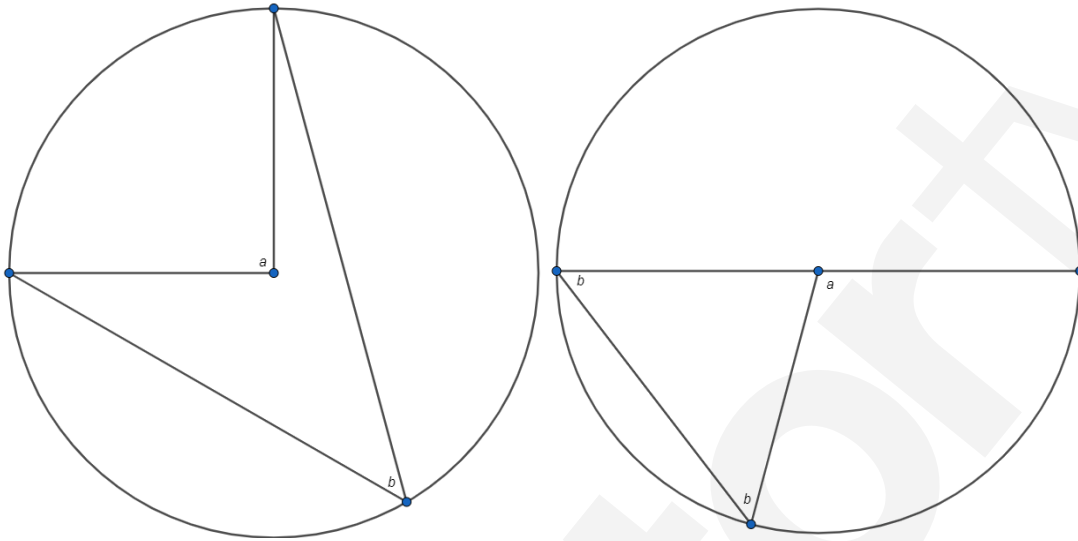
- For any sector in a circle with a central angle (in radians) of  $\theta$ , where  $r$  is the radius of the circle, and the area  $A$  of that circle:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

This simplifies to a shorter equation, which may merit memorizing:

$$A = \frac{1}{2} \theta r^2$$

- Inscribed Angle Theorem: States that an angle (portrayed as  $b$  below) inscribed in a circle is equal to half of the central angle (portrayed as  $a$  below) that subtends the same arc. Both scenarios below show the central angle theorem:



- **Trigonometry:**

SOHCAHTOA: For any angle  $\theta$  in a right triangle,  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ ,  $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$   
and  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ .

**Passport to Advanced Math:**

- **Function Notation:** (many of these are unlikely to appear, but you should understand how function notation works)
- $(f \circ g)(x)$  means  $f(g(x))$
- $(f * g)(x)$  and  $fg(x)$  mean  $f(x) * g(x)$
- $\left(\frac{f}{g}\right)(x)$  means  $\frac{f(x)}{g(x)}$

**Problem Solving and Data Analysis:**

**General Statistics:**

- Average: Sum of terms divided by number of terms
- Median: The numerical middle value of a set of terms
- Mode: The most common term in a set of terms

**Graphs/Plots:**

- **Line of Best Fit:** A line that best represents the data on a scatterplot. It can be used to estimate the value of points not on the plot itself. **TIP:** pluck points and use a calculator program to find  $y=mx+b$  on scatterplots!

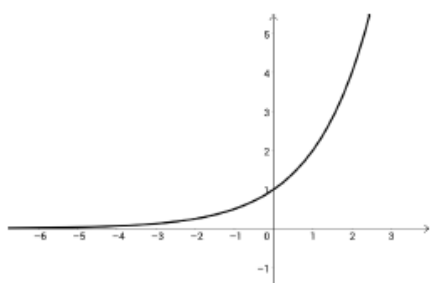
## GRAPH BEHAVIOR:

### Types of graphs:

**Graph of an exponential function:**

Standard Form:  $f(x) = a^x$

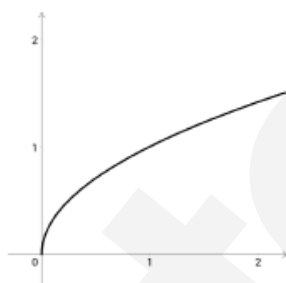
Example:  $y = 2^x$



**Graph of a square root:**

Standard Form:  $y = \sqrt{x}$

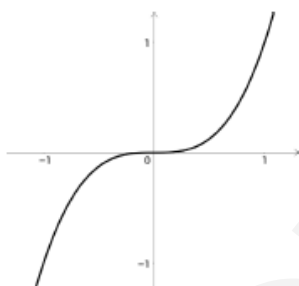
Example (Parent Graph):  $y = \sqrt{x}$



**Graph of a cubic function:**

Vertex Form:  $y = a(x - h)^3 + k$

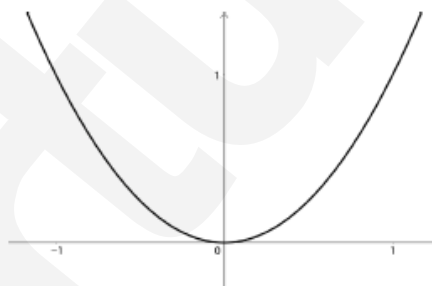
Example (Parent Graph):  $y = x^3$



**Graph of a parabola (quadratic function):**

Vertex Form:  $f(x) = a(x - h)^2 + k$

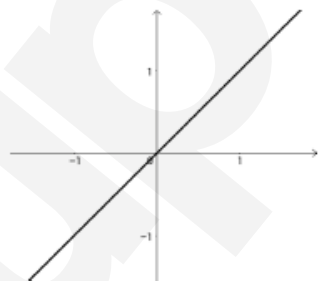
Example (Parent Graph):  $y = x^2$



**Graph of a linear equation:**

Slope-Intercept Form:  $y = mx + b$

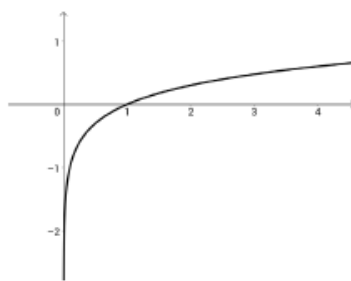
Example (Parent form):  $y = x$



**Graph of a logarithmic function:**

Standard Form:  $y = \log_a x$

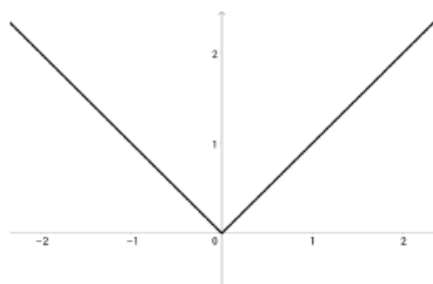
Example:  $y = \log_{10} x$



**Graph of an absolute value function:**

Vertex Form:  $y = a|x - h| + k$

Example (Parent Graph):  $y = |x|$



- Horizontal Shift: For ALL functions, if you replace all instances of  $x$  in a function with " $(x - h)$ " you'll find that the graph moves " $h$ " units to the right.
- Vertical Shift: If you replace all instances of  $y$  in a function with " $(y - k)$ " you'll find that the graph moves " $k$ " units upward.

## END BEHAVIOR:

| <b>Degree: Even</b>                                                                 | <b>Degree: Even</b>                                                                   |
|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| Leading Coefficient: positive                                                       | Leading Coefficient: negative                                                         |
| End Behavior: $f(x)$ approaches $+\infty$ at both ends of the graph (upward facing) | End Behavior: $f(x)$ approaches $-\infty$ at both ends of the graph (downward facing) |
| Domain: all reals                                                                   | Domain: all reals                                                                     |
| Range: all reals $\geq$ maximum                                                     | Range: all reals $\leq$ maximum                                                       |
| Example: $y = x^2$                                                                  | Example: $y = -2x^6 + 3x^5 + 4x$                                                      |

| <b>Degree: odd</b>                                                                                                    | <b>Degree: odd</b>                                                                                                      |
|-----------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| Leading Coefficient: positive                                                                                         | Leading Coefficient: negative                                                                                           |
| End Behavior: At graph left, $f(x) \rightarrow -\infty$ . At graph right, $f(x) \rightarrow +\infty$ (upward sloping) | End Behavior: At graph left, $f(x) \rightarrow +\infty$ . At graph right, $f(x) \rightarrow -\infty$ (downward sloping) |
| Domain: all reals                                                                                                     | Domain: all reals                                                                                                       |
| Range: all reals                                                                                                      | Range: all reals                                                                                                        |
| Example: $f(x) = x^3$                                                                                                 | Example: $f(x) = -3x^5 - 2x$                                                                                            |