# SAT MATH SECTION: CHEAT SHEET
# FORMULAS, RULES, AND DEFINITIONS

**Note: important formulas included at beginning of every test not included below**

## ALGEBRA:

ZERO PRODUCT PROPERTY: If $ab = 0$, then $a = 0$, $b = 0$, or $a = b = 0$.

SYSTEMS OF EQUATIONS:
When you solve down a system of equations, one of three outcomes is possible as you run the algebra and create a final, simplified equation:

| **One solution** | **Zero solutions** | **Infinite solutions** |
|---|---|---|
| If you get a single x or y value, (i.e. x=5 or y=2) you have one solution | If you get a statement that is never true (i.e. 5=9 or 0=4), you have no solutions. | If you get two values that always equal each other (i.e. 3x=3x or 5=5), you have infinite solutions. |
| **Comparing equations:** You can alternatively put EVERY equation in slope intercept form that is part of your system of linear equations. Then look at each of the y=mx+b forms and compare the two slopes (i.e. the “m’s”) and y-intercepts (the “b’s”). | | |
| **One solution** | **Zero solutions** | **Infinite solutions** |
| Different slopes (intercepts don’t matter) | Same slopes (parallel lines), different y-intercepts | Same slope, same intercept i.e. identical equations |

FOIL AND FACTORING:

- A **monomial** is a single product such as $4x, 7x^3$, or $8n^2$.
- A **binomial** has two elements added together such as $4x+3$ or $5n^3+3n$.
- A **polynomial** has multiple elements added together such as $5n^3+3n^2+7n+2$ or $5n^3+3n^2+7n+2$ or $5x^2+2x+4$ $5x^2+2x+4$.
- Difference of Squares: The product of the difference $(a-b)$ and the sum $(a+b)$ is equal to $a$ squared minus $b$ squared, $(a-b)(a+b) = a^2 - b^2$
- Square of a Sum: $(a+b)^2 = a^2 + 2ab + b^2$
- Square of a Difference: $(a-b)^2 = a^2 - 2ab + b^2$

SLOPE FORMULA: For points $(x_1, y_1)$ and $(x_2, y_2)$, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

SLOPE-INTERCEPT FORM: $y = mx + b$
Where $m$ is the **slope** of the line, and $b$ is the **y-intercept** of the line at point $(0, b)$.

MIDPOINT FORMULA: The midpoint of two coordinate points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

$$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

DISTANCE FORMULA: Given two points, $(x_1, y_1)$ and $(x_2, y_2)$, the distance between them is:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

SPEED AND RATES DISTANCE FORMULA: $d = rt$
WORK FORMULA: $w = rt$
COMBINED WORK FORMULA: $w = (r_1 + r_2)t$

QUADRATICS AND POLYNOMIALS: Important- know how to find the vertex of ANY parabola!

- VERTEX FORM OF A QUADRATIC: the vertex form of a parabola is: $f(x) = a(x - h)^2 + k$
  - The vertex of the parabola in this form is $(h, k)$
  - When $a$ is positive, the parabola opens upwards, and the minimum is $(h, k)$.
  - When $a$ is negative, the parabola opens downwards, and the minimum is $(h, k)$.
  - Alternate form for parabolas with horizontal axis (RARE, unlikely to appear) $x = a(y - k)^2 + h$, positive $a$ opens right-ward, negative $a$ opens leftward

- FACTORED FORM: The factored form of a polynomial usually takes the form: $f(x) = a(x - n)(x - m)$
  - When $a$ is positive, the parabola opens upwards.
  - When $a$ is negative, the parabola opens downwards.
  - The graph crosses the x-axis at each point when a factored piece equals zero (the zeros). For example, at x=n, x-n equals 0, so the graph crosses through the point (n, 0) on the x-axis.

  In the case of a quadratic, which will typically have two roots (i.e. two factored pieces that include an x), the midpoint of 2 zeros is the x-value of the vertex. In the example above, the zeros are at x=n and x=m. So the x value of its vertex is the average of n and m.

- STANDARD FORM: the standard form of the parabola has the general form: $f(x) = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$
  - $-\frac{b}{2a}$ is the x-value of the vertex. The y-value of the vertex can be found by plugging in this value for $x$ and solving for $y$ (or $f(x)$). Memorize this!
  - Assuming a vertically oriented parabola, the vertex is always either the maximum or the minimum of the graph. (remember max/min is always the y-value!)
  - When $a$ is positive, the parabola opens upwards.

- When $a$ is negative, the parabola opens downwards.
- The sum of the two roots is $-\frac{b}{a}$ (not necessary to know, but occasionally helpful)
- The product of the two roots is $\frac{c}{a}$ (not necessary to know, but occasionally helpful)

- QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Memorize this. Also, we recommend you program it into your calculator. (See TI 84 programs for download in the online course)

- FINDING SOLUTIONS USING THE DISCRIMINANT: Given that $f(x) = ax^2 + bx + c$, the discriminant is defined as $b^2 - 4ac$ (the piece under the root in the quadratic equation):
  - When this value is positive, there are two real roots
  - When this value is 0, there is one real root
  - When this value is negative, there are no real roots but two imaginary roots

RADICAL AND EXPONENT RULES:

| EXPONENT RULES | RADICAL RULES |
|---|---|
| Zero Power Rule: $a^0 = 1$ | Fractional exponent conversion: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$ |
| Negative exponent rule: $a^{-b} = \frac{1}{a^b} a$ | Product of radicals: $\sqrt[c]{a}\sqrt[c]{b} = \sqrt[c]{ab}$ |
| Power of a power: $\left(a^b\right)^c = a^{bc}$ | |
| Product of powers: $(ab)^c = a^{b+c}$ | |
| Power of a product: $(ab)^c = a^c b^c$ | |
| Power of a quotient: $\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$ | |
| Quotient of powers: $\frac{a^b}{a^c} = a^{b-c}$ | |

THE DEFINITION OF A LOGARITHM: $\log_c a = b$ means that $c^b = a$

COMMON LOGARITHMS: $\log x = \log_{10} x$

NATURAL LOGARITHMS: $\ln x = \log_e x$

CHANGE OF BASE FORMULA: For all positive numbers a , b , and c, where $b \neq 1$ and $c \neq 1$:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

CIRCLE THEOREMS:

- Circle Equation: $(x-h)^2+(y-k)^2=r^2$ with center point at $(h,k)$

**Angle to Arc Length Equations:** Central angles are proportional to the arc length they intercept, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{central\ angle\ measure}{total\ measure\ of\ angle\ in\ a\ circle} = \frac{arc\ length}{total\ circumference}$$

**DEGREES VERSION OF EQUATION:**

- More specifically, this means if a circle has a central angle a, radius r, and arc length L:

$$\frac{a^\circ}{360^\circ} = \frac{L}{2\pi r}$$

**RADIANS VERSION OF EQUATION:**

- For any arc on a circle with a central angle (in radians) of $\theta$, where $r$ is the radius of the circle, the length $L$ of the arc is given by:

$\frac{\theta}{2\pi}=\frac{L}{2\pi r}$ which simplifies to a short handy equation that's a good idea to memorize: $L=\theta r$ .

**Angle to Sector Area Equations:** Central angles are proportional to the area of the sector they include, so you can create a proportion that equates part over whole equal to part over whole:

$$\frac{central\ angle\ measure}{total\ measure\ of\ angle\ in\ a\ circle} = \frac{\text{sector area}}{\text{total circle area}}$$

**DEGREES VERSION OF EQUATION:**

- For any sector on a circle with a central angle (in degrees) of $a$, where $r$ is the radius of the circle, and the area is $A$:

$$\frac{a^\circ}{360^\circ} = \left(\frac{A}{\pi r^2}\right).$$

**RADIANS VERSION OF EQUATION:**

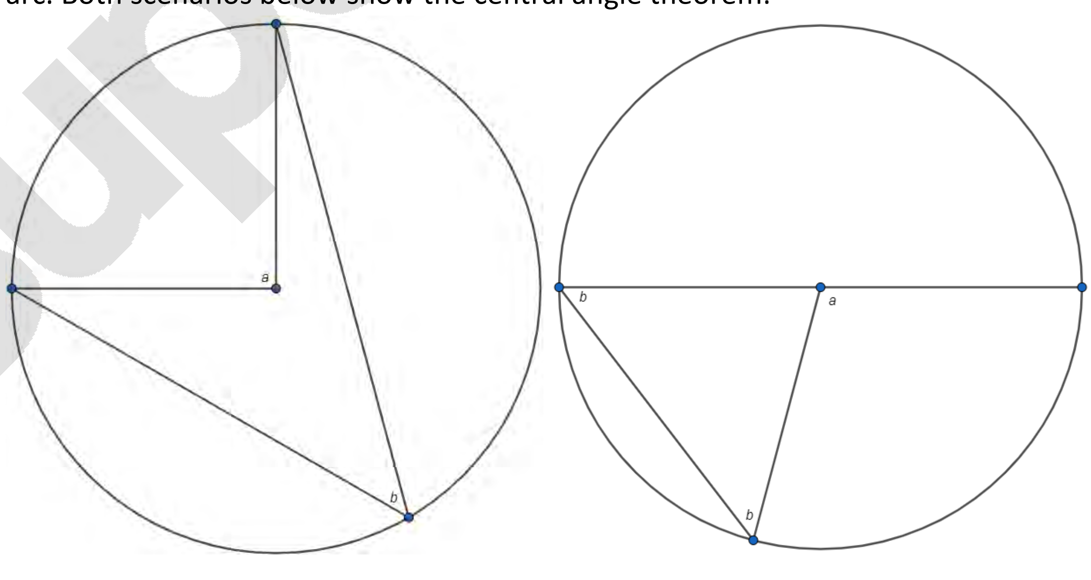
- For any sector in a circle with a central angle (in radians) of $\theta$, where $r$ is the radius of the circle, and the area $A$ of that circle:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

This simplifies to a shorter equation, which may merit memorizing:

$$A = \frac{1}{2}\theta r^2$$

- Inscribed Angle Theorem: States that an angle (portrayed as $b$ below) inscribed in a circle is equal to half of the central angle (portrayed as $a$ below) that subtends the same arc. Both scenarios below show the central angle theorem:



- **Trigonometry:**

  SOHCAHTOA: For any angle $\theta$ in a right triangle, $\sin\theta = \frac{opposite}{hypotenuse}$ , $\cos\theta = \frac{adjacient}{hypotenuse}$ and $\tan\theta = \frac{opposite}{hypotenuse}$.

**Passport to Advanced Math:**

- **Function Notation:**
- $(f \circ g)(x)$ means $f(g(x))$
- $(f * g)(x)$ and $fg(x)$ mean $f(x) * g(x)$
- $\left(\frac{f}{g}\right)(x)$ means $\frac{f(x)}{g(x)}$

**Problem Solving and Data Analysis:**

- **General Statistics:**
- Average: Sum of terms divided by number of terms
- Median: The numerical middle value of a set of terms
- Mode: The most common term in a set of terms
- **Graphs/Plots:**
- Line of Best Fit: A line that best represents the data on a scatterplot. It can be used to estimate the value of points not on the plot itself.

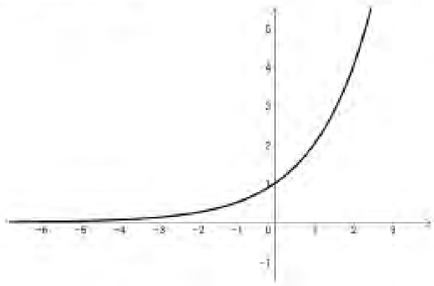
## GRAPH BEHAVIOR:

Types of graphs:

**Graph of an exponential function:**

Standard Form: $f(x)=a^x$

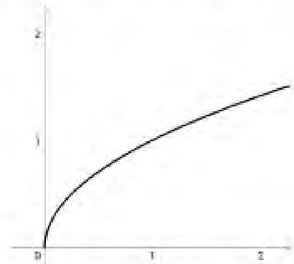
Example: $y=2^x$



**Graph of a square root:**

Standard Form: $a\sqrt{x}$

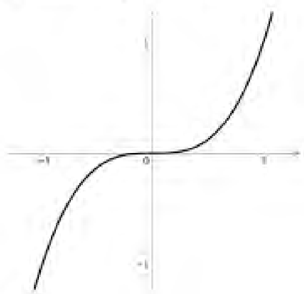
Example (Parent Graph): $y=\sqrt{x}$



**Graph of a cubic function:**

Vertex Form: $y=a(x-h)^3+k$

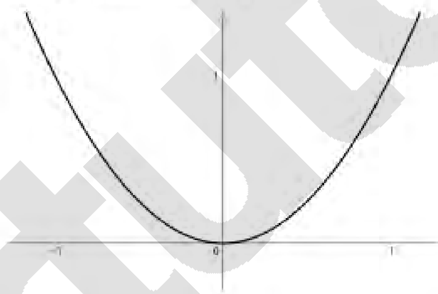
Example (Parent Graph): $y=x^3$



**Graph of a parabola (quadratic function):**

Vertex Form: $f(x)=a(x-h)^2+k$

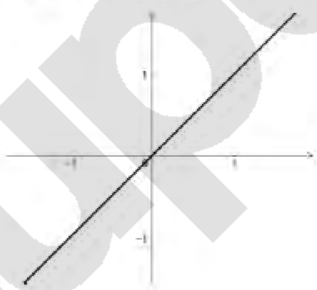
Example (Parent Graph): $y=x^2$



**Graph of a linear equation:**

Slope-Intercept Form: $y=mx+b$

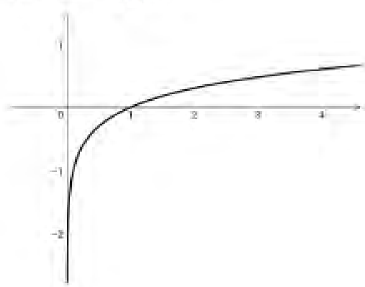
Example (Parent form): $y=x$



**Graph of a logarithmic function:**

Standard Form: $y=\log_a x$

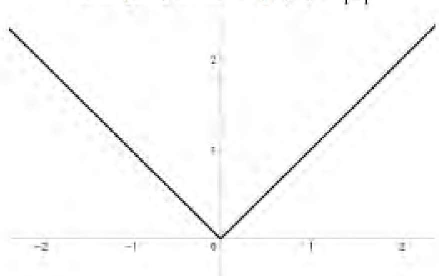
Example: $y=\log_{10} x$



**Graph of an absolute value function:**

Vertex Form: $y=a|x-h|+k$

Example (Parent Graph): $y=|x|$



- Horizontal Shift: For ALL functions, if you replace all instances of x in a function with "$(x-h)$" you'll find that the graph moves "h" units to the right.
- Vertical Shift: If you replace all instances of y in a function with "$(y-k)$" you'll find that the graph moves "k" units upward.

END BEHAVIOR:

| **Degree: Even** | **Degree: Even** |
|---|---|
| Leading Coefficient: positive | Leading Coefficient: negative |
| End Behavior: $f(x)$ approaches $+\infty$ at both ends of the graph (upward facing) | End Behavior: $f(x)$ approaches $-\infty$ at both ends of the graph (downward facing) |
| Domain: all reals | Domain: all reals |
| Range: all reals ≥ maximum | Range: all reals ≤ maximum |
| Example: $y=x^2$ | Example: $y=-2x^6+3x^5+4x$ |

| **Degree: odd** | **Degree: odd** |
|---|---|
| Leading Coefficient: positive | Leading Coefficient: negative |
| End Behavior: At graph left, $f(x)\rightarrow-\infty$. At graph right, $f(x)\rightarrow+\infty$ (upward sloping) | End Behavior: At graph left, $f(x)\rightarrow+\infty$ . At graph right, $f(x)\rightarrow-\infty$ (downward sloping) |
| Domain: all reals | Domain: all reals |
| Range: all reals | Range: all reals |
| Example: $f(x)=x^3$ | Example: $f(x)=-3x^5-2x$ |