ACT MATH SECTION: FORMULAS, RULES, AND DEFINITIONS

ALGEBRA:

ZERO PRODUCT PROPERTY: If $ab = 0$, then $a = 0$, $b = 0$, or $a = b = 0$.

SYSTEMS OF EQUATIONS:

One solution	**Zero solutions**	**Infinite solutions**
If you get a single x or y value, you have one solution	If you get a statement that is never true, you have no solutions.	If you get two values that always equal each other, you have infinite solutions.
Comparing slopes of equations:		
Different slopes (intercepts don't matter)	Same slopes (parallel lines), different y-intercepts	Same slope, same intercept

FOIL AND FACTORING:

- A **monomial** is a single product such as $4x, 7x^3$, or $8n^2$.
- A **binomial** has two elements added together such as $4x+3$ or $5n^3+3n$.
- A **polynomial** has multiple elements added together such as $5n^3+3n^2+7n+2$ or $5n^3+3n^2+7n+2$ or $5x^2+2x+4$ $5x^2 + 2x + 4$.
- Difference of Squares: The product of the difference $(a-b)$ and the sum $a+b$ is equal to a squared minus b squared, $(a-b)(a+b) = a^2 - b^2$ $(a-b)(a+b) = a^2 - b^2$
- Square of a Sum: $(a+b)^2 = a^2 + 2ab - b^2$
- Square of a Difference: $(a-b)^2 = a^2 - 2ab - b^2$ $(a-b)^2 = a^2 + 2ab - b^2$

SLOPE FORMULA: For points (x_1, y_1) and (x_2, y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$.

SLOPE-INTERCEPT FORM: $y = mx + b$
Where m is the **slope** of the line, and b is the **y-intercept** of the line at point $(0, b)$.

MIDPOINT FORMULA: The midpoint of two coordinate points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$$

$$\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$$

DISTANCE FORMULA: Given two points, (x_1, y_1) and (x_2, y_2), the distance between them is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

SPEED AND RATES DISTANCE FORMULA: $d = rt$

WORK FORMULA: $w = rt$
COMBINED WORK FORMULA: $w = (r_1 + r_2)t$ $w = (r_1 + r_2)t$

PARABOLA FORMULA: $y = a(x-h)^2 + k$ OR $x = a(y-k)^2 + h$ $y = a(x-h)^2 + k$
$x = a(y-k)^2 + h$

QUADRATICS AND POLYNOMIALS:

- VERTEX FORM: the vertex form of a parabola is: $f(x) = a(x-h)^2 + k$
  - The vertex of the parabola in this form is (h, k)
  - When a is positive, the parabola opens upwards, and the minimum is (h, k).
  - When a is negative, the parabola opens downwards, and the minimum is (h, k).
- FACTORED FORM: The factored form of a polynomial usually takes the form: $f(x) = a(x-n)(x-m)$ $f(x) = a(x-n)(x-m)$
  - When a is positive, the parabola opens upwards.
  - When a is negative, the parabola opens downwards.

  The midpoint of 2 zeros is the x-value of the vertex, with zeros at values of x such that—
  $(ax - m) = 0$
  $(ax - n) = 0$
- STANDARD FORM: the standard form of the parabola has the general form: $f(x) = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$
  - $-\frac{b}{2a}$ is the x-value of the vertex. The y-value of the vertex can be found by plugging in this value for x and solving for y (or $f(x)$).
  - The vertex is always either the maximum or the minimum of the graph.
  - When a is positive, the parabola opens upwards.
  - When a is negative, the parabola opens downwards.
  - The sum of the two roots is $-\frac{b}{a}$ (not necessary to know)
  - The product of the two roots is $\frac{c}{a}$ (not necessary to know)

- QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- FINDING SOLUTIONS USING THE DISCRIMINANT: Given that $f(x) = ax^2 + bx + c$, the discriminant is defined as $b^2 - 4ac$ $b^2 - 4ac$ (the argument of the root in the quadratic equation):
  - When this value is positive, there are two real roots
  - When this value is 0, there is one real root
  - When this value is negative, there are no real roots (but it does have two imaginary roots)

RADICAL AND EXPONENT RULES:

EXPONENT RULES	RADICAL RULES
Zero Power Rule: $a^0 = 1$	Fractional exponent conversion: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$
Negative exponent rule: $a^{-b} = \frac{1}{a^b} a$	Product of radicals: $\sqrt[c]{a}\sqrt[c]{b} = \sqrt[c]{ab}$
Power of a power: $\left(a^b\right)^c = a^{bc}$	
Product of powers: $(ab)^c = a^{b+c}$	
Power of a product: $(ab)^c = a^c b^c$	
Power of a quotient: $\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$	
Quotient of powers: $\frac{a^b}{a^c} = a^{b-c}$	

THE DEFINITION OF A LOGARITHM: $\log_c a = b$ means that $c^b = a$

COMMON LOGARITHMS: $\log x = \log_{10} x$

NATURAL LOGARITHMS: $\ln x = \log_e x$

CHANGE OF BASE FORMULA: For all positive numbers a , b , and c, where $b \neq 1$ and $c \neq 1$:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

PROPERTIES OF LOGARITHMS:

- Power Property: $a \log x = \log_b x^a$
- Product Property: $\log_a x + \log_a y = \log_a xy$
- Quotient Property: $\log_a x - \log_a y = \log_a \frac{x}{y}$

DEFINITION OF LOGARITHMS:

- $n^{\log_n} = a$
- $\log_x x^n = n \log_x x^n = n \log$
- Logarithm of the base: $\log_x x = 1 \log_x x = 1$

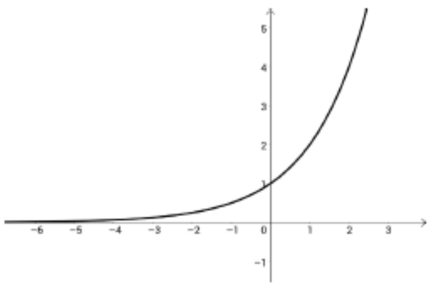
CONICS:

- Circle Equation: $(x-h)^2 + (y-k)^2 = r^2$ with center point at (h, k)
- Ellipse Equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ with center point at (h, k) and the distance from that center to the end points of the major and minor axes denoted by a and b .

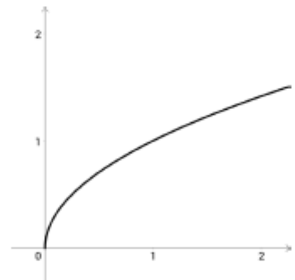
- Equation of a Hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$
- Conic Equation Types: A conic section of the form, $Ax^2+By^2+Cx+Dy+E=0$ in which A and B are both not zero is:
    - A circle if A =B .
    - A parabola if AB = 0.
    - An ellipse if A ≠B and AB >0.
    - A hyperbola if AB <0.

GRAPH BEHAVIOR:
Types of graphs:

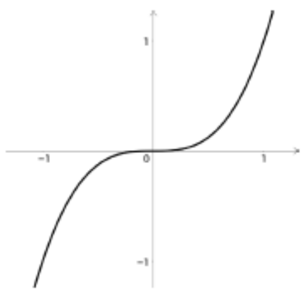
Graph of an exponential function:
Standard Form: $f(x) = a^x$
Example: $y = 2^x$



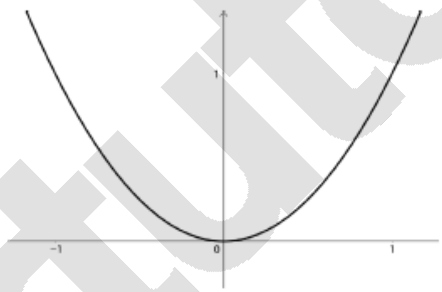
Graph of a square root:
Standard Form: $a\sqrt{x}$
Example (Parent Graph): $y = \sqrt{x}$



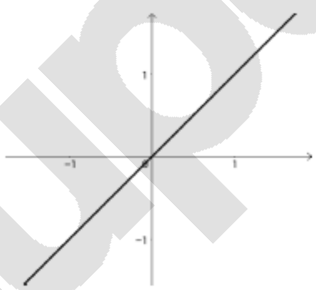
Graph of a cubic function:
Vertex Form: $y = a(x-h)^3 + k$
Example (Parent Graph): $y = x^3$



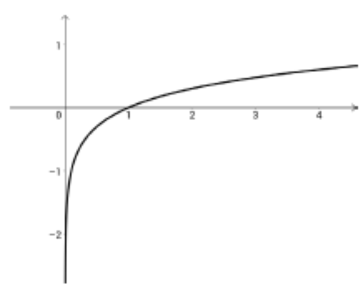
Graph of a parabola (quadratic function):
Vertex Form: $f(x) = a(x-h)^2 + k$
Example (Parent Graph): $y = x^2$



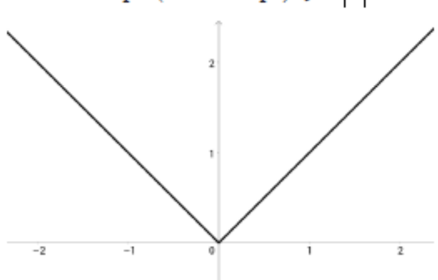
Graph of a linear equation:
Slope-Intercept Form: $y = mx + b$
Example (Parent form): $y = x$



Graph of a logarithmic function:
Standard Form: $y = \log_a x$
Example: $y = \log_{10} x$



Graph of an absolute value function:
Vertex Form: $y = a|x-h| + k$
Example (Parent Graph): $y = |x|$



- Horizontal Shift: For ALL functions, if you replace all instances of x in a function with "$(x-h)$" you'll find that the graph moves "h" units to the right.
- Vertical Shift: If you replace all instances of y in a function with "$(y-k)$" you'll find that the graph moves "k" units upward.

END BEHAVIOR:

Degree: Even	**Degree: Even**
Leading Coefficient: positive	Leading Coefficient: negative
End Behavior: $f(x)$ approaches $+\infty$ at both ends of the graph (upward facing)	End Behavior: $f(x)$ approaches $-\infty$ at both ends of the graph (downward facing)
Domain: all reals	Domain: all reals
Range: all reals ≥ maximum	Range: all reals ≤ maximum
Example: $y=x^2$	Example: $y=-2x^6+3x^5+4x$

Degree: odd	**Degree: odd**
Leading Coefficient: positive	Leading Coefficient: negative
End Behavior: At graph left, $f(x)\to-\infty$. At graph right, $f(x)\to+\infty$ (upward sloping)	End Behavior: At graph left, $f(x)\to+\infty$. At graph right, $f(x)\to-\infty$ (downward sloping)
Domain: all reals	Domain: all reals
Range: all reals	Range: all reals
Example: $f(x)=x^3$	Example: $f(x)=-3x^5-2x$

TRANSLATIONS AND REFLECTIONS:

- Reflectional symmetry: the property a figure has if half of the figure is congruent to the other half over an axis.
- Rotational symmetry: also known as radial symmetry—the property a figure has if it is congruent to itself after some rotation less than 360°.